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The Aesthetics of Visual Form

The Golden Section Rectangle Enhances Depth Perception

Abstract: A perennial question involves the relation between linguistic structures and processes in other modalities, such as vision. Current investigations of linguistic universals have begun to explore formal/physical constraints on linguistic universals, such as the impact of the Fibonacci series on the ideal branching structure of phrase hierarchies. The mathematical limit of the Fibonacci series is the Golden Mean Ratio (ϕ = 1.618...:1). In this study, we propose and test a new theory of the human preference for the golden rectangle – one of the oldest problems of psychology. The initial visual decomposition of the golden rectangle into primitives elicits representational processes that explain its preference, according to classic Aristotelian aesthetic theories. The same decomposition processes involve a unique initial access of the third dimension. Thus, we predict that the golden rectangle uniquely enhances depth perception of scenes within it. Several new experiments confirm this prediction, which also explains our discovery that some artist schools systematically used the golden rectangle to frame topographic landscapes. The broader implication of this finding is support for the pervasive relevance of natural laws as part of a wide range of human cognitive behaviors.

Keywords: aesthetics; depth perception; golden mean; language; vision

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1 Aesthetics, the Golden Mean, and Language Science

This paper reviews several cognitive/perceptual theories of aesthetics that offer an explanation as to why the golden mean ratio of lengths has often been found to be preferred by humans over other ratios. We end by experimentally demonstrating an unexpected prediction of the explanations, namely that golden mean ratio frames enhance the illusion of depth of stimuli within them.

First, we need to set the scene as to why the sciences of language can benefit from considerations of aesthetic experiences in general and the golden mean ratio in particular. We organize this scene-setting in terms of a set of questions and brief answers.

1.1 Why Should Linguistics Be Interested in Aesthetic Experiences?

One of the great puzzles of language acquisition is what drives the individual child to untangle the many abstract puzzles that each language presents: the child's linguistic experience is a puzzle in part because of the well-known poverty of the stimulus around himer (Chomsky, 1987; Fodor & Crowther 2002), e.g., false starts, ungrammatical sentences, swallowed words, erratic error correction, etc. In part it is a puzzle because even with complete and error free sentences to experience, and perfect error correction by the child's environment, the number of experienced sentences is very small and underdetermines the grammar that the child acquires: no matter what one's particular theory of what a structural grammar is, the child must bring to the table considerable statistical and symbolic acumen, along with a persistent motivation to use hiser limited input to arrive at grammatical competence. Thus the acquisition problem is not only the "poverty of the stimulus", it is "the mystery of motivation".

A common answer to both problems is that the essential features of language are innately available in such a way that language automatically grows out of the combination of biologically available innate structures and how they organize haphazard experiences by sensitivity to highly specific cues: there is no motivation such as the need to communicate or the need to compile habits (Fodor & Crowther, 2002; Hauser et al., 2002). This approach has created a theoretical rift in theories of acquisition between two long-standing approaches to cognition and learning: i) everything we do is based on habits (statistical learning); ii) everything important that we do is based on structural computations.

There is a tradition within the literature on acquiring a structure like language that has suggested a middle way, integrating the learning of habits with the discovery of structure (e.g. Sinclaire de Zwart, 1973; Karmiloff-Smith & Inhelder, 1974.) On these views, learning involves the critical role of statistical information in differentiating potential mental hypotheses. Recently, Bever has argued that this dual process is in fact typical of problem solving in general, as suggested for example by the gestalt psychologists (see Bever, 2012 for a review; and Bever, 1987, 2008 for relevant discussions; Wertheimer, 1945 for a presentation of problem solving). In the gestalt view, a real problem is one that involves a conflict between two ways of looking at a situation: solving it creatively involves finding a third way that resolves the initial conflict. The gestalt psychologists are credited with the notion that solving a problem elicits an "aha" reaction, an emotionally exciting surge of mental energy, which is consciously released when someone finds a solution.

Thus, combining the two traditions, we can suggest that learning the intricate grammar of a language is akin to solving a problem – it involves the child accumulating statistically repetitive language patterns: but then when a pattern fails, it creates a mental problem which the child attempts to solve by accessing hiser available computational capacities to create a new model of hiser language that accommodates both the patterns and the apparent exceptions. A much discussed example of this is the accumulation of the canonical form of a language e.g., Noun + Verb + Noun = agent + action + object in English, and the reanalysis that occurs when stimulated by constructions that violate this, e.g., the object relative, or the passive.

So, succinctly put: figuring out the patterns of one's language involves normal problem solving processes: in solving explicit problems finding a solution is recognizably rewarding to humans. So, possibly learning the language patterns is rewarding to children in itself.

We are not finished however, which is what leads us to consideration of aesthetically satisfying experiences. The Gestaltists were addressing explicit solving of explicit problems with conscious recognition of excitement. But no one tells a child that s/he has to solve the language riddle and there may be no recognizable "aha" when the child figures out a bit of hiser grammar. Rather, grammar learning may be intrinsically rewarding in a series of mini-"aha" experiences, exciting enough to keep motivating the child, but not so obvious as to engage self recognition as having figured out a bit of the language patterns.

How can we study the concept of unconscious problem solving experiences and which ones are preferred? The study of everyday aesthetic experiences can offer a possible domain: as Gustav Fechner, the nineteenth century pioneer of aesthetics and judgment put it, when we study aesthetic preferences, we study the operation of the mind when it "runs freely", unconstrained by practical or externally functional constraints (Freidin & Vergnaud, 2001). For our purposes, the study of everyday aesthetic objects may show how to unify unconscious problem solving processes with motivation – that is, the intrinsic motive to prefer some kinds of stimuli over others may be because the preferred stimuli elicit unconscious problem solving processes of the same kind as elicited in explicit problem solving.

1.2 Why Should We Concentrate on the Golden Mean?

We have referred to "everyday" aesthetics in order to differentiate them from the fine arts. The latter are suffused with cultural history, fashion, money, and explicit creativity: thus, they do not satisfy Fechner's criteria in a simple way – they may often involve real mental aesthetic processes, but obscured by many other constraints they must satisfy (but see Lasher et al., 1983). But there are candidates for objects that appear and re-appear in many cultures and times. A simple example is the mother-infant game of peek-a-boo – on one analysis it is enjoyable because it plays off the representational conflict between the present, but then absent mother, to be resolved by the "return" of the present mother, mediated by the abstract concept that when the mother disappears, she still exists. Another example is the pervasive rhythmic tattoo associated in English with "shave and a haircut, two bits": in fact, it has two potential underlying competing rhythmic time signatures, the first being 2/4, and the other as 3/4, being associated with "shave and a massage, two bits" which are reconciled at the final beat ("bits").

These examples of intuitively preferred objects exist along with others, including the preference for the golden mean, which do meet Fechner's criterion for an item that reveals mental processes unfettered by practical constraints. But the golden mean also has characteristics that have been investigated in the study of how mathematical/physical laws can appear in language universals. Recent language investigations have suggested that as the limit of the Fibonacci series, it appears in the architecture of language in various ways: i) in footing syllables (Idsardi, 2008; Idsard & Uriagereka, 2009), ii) as the best mediator between serial and derivational processes and constraints (Uriagereka & Piattelli-Palmarini, citing Townsend & Bever, 2001, among others), iii) X-bar molecule of phrase growth is best structural compromise between multiple branching from single node, and one terminal per node (spine) (Medeiros & Piattelli-Palmarini, in press).

1.3 Why Should Linguistics be Interested in Structural Parallels in Vision?

First, as a natural behavior, which is not explicitly taught to children, visual processing may share certain universal mental/cognitive/perceptual principles with language: thus processes shared by language and vision may reveal deep properties of brain and mind. Second, more narrowly, we may find in vision certain particular cognitive universals that bear on linguistic universals. Finally, we know that language has taken over brain areas that originally have a long evolutionary history of adaptation or functional specificity for vision – hence some computational processes evolved for vision may be carried over as fundamental to linguistic computations. (See references in the previous paragraph, and Bever, 2012 for elaboration).

2 The Empirical Study of the Golden Mean Rectangle and Depth Perception

Artists have choices in how they frame their work, as well as what content to depict. Nineteenth century American painters emphasized the unique expressiveness of American landscapes for both political and aesthetic reasons (Baratte, 1976; Wilmerding, 1976). We have discovered that American painters of this era also differentiated the frame proportions of their canvases according to the scene content (see Table 1). We classified paintings (Benjafield, 1985; Benjafield & Davis, 1978; Benjafield & Adams-Webber, 1975, 1976; Webber & Rodney, 1983; Manoogian, 1989) into three types: paintings that contained landscapes that emphasized depth; paintings that contained landscapes, but did not emphasize depth; and non-landscape paintings, such as portraits and interiors. The distinction between the first two is crucial, because we wished to test if artists use of frame shapes differed when the illusion of a third dimension (depth) was intended. This distinction was scored by an art historian familiar with art of this period but not familiar with our hypothesis *(*Ayres, 1986*)*. Landscapes were framed using proportions of 1.6:1, while interiors and portraits were framed using proportions of 4:3.[1, 2]

Table 1: The frame proportions of 19th century American paintings (N = 145) as a function of the painting's content

Painting classification	Mean length:width ratio	95% confidence interval
Landscapes: depth emphasis	1.63	± 0.07 (n = 26)
Landscapes: no depth emphasis	1.44	± 0.07 (n = 91)
Portraits and interiors	1.31	± 0.06 (n = 28)

Why did American painters choose the 1.6:1 frame proportion to depict expansive landscapes? This proportion is, in fact, the *golden section* proportion, which Gustav Fechner (1897), founder of the fields of psychophysics and empirical aesthetics, and later researchers (Benjafield, 1976; Piehl, 1978; McManus, 1980; Green, 1995; Benjafield & McFarlane, 1997; Fechner et al., 1997) have shown to be the consistently preferred rectangular shape. In this paper, we propose and demonstrate an explanation of the aesthetic preference for this proportion and its selective use by nineteenth century American painters. We combine Aristotelian aesthetic theories (optimal complexity and conflict resolution) and visual-form perception theory (decomposition into components) to hypothesize that golden section rectangles elicit initial visual representations in two dimensions that stimulate three-dimensional representations. We show that this perceptual process correctly predicts enhanced depth perception within golden section frames compared to frames of other proportions (Bever, 1987). This is the first report of the influence of frame shape on depth perception, the first demonstration that the golden section rectangle enhances depth perception, and the first demonstration grounded in modern visual theories of *why* the golden section rectangle is aesthetically satisfying.

The *golden rectangle* (Figure 1A; see Appendix for all Figures) can be constructed from a line divided by the golden section as defined by dividing a line into short (S) and long (L) segments such that:

1 British artists showed a similar trend for both kinds of paintings; however, French artists of the same period did not differentiate frame shape by content, using 1.33:1 for both landscapes and non-landscapes. These differences may result from cultural factors governing the value of painting. In particular, French paintings were constrained by the standard shape that was eligible for shows and private salons.

2 These new observations refine a previous survey of all frame shapes, which did not distinguish the painting's content (Shortess et al., 1997).

(1) $$\frac{S}{L} = \frac{L}{S+L}$$

If the longer segment (L) is of length 1, then the length of the whole line $S+L$ is an irrational number, ϕ (phi), with a value of 1.618.... Similarly, if $S+L$ is of length 1, then the *golden section* (L) is equal to ϕ-1 (0.618...).

ϕ□is a unique constant with many properties that have captured the interest of mathematicians and writers for several thousand years including current fiction and popular science (Livio, 2002).[3] It bridges geometric (multiplicative) and arithmetic (additive) growth, and is unique in being the only number that is the root of unity and itself (ad infinitum), and in being the only number which is unity larger than its own reciprocal (also ad infinitum).[4]□ ϕ□is a transcendental constant (Weisstein, 2003a) and thus, like π□and e, is irrational; indeed, ϕ is unique in being the "most" irrational of numbers (Shallit, 1975). While irrational, ϕ is also

3 The earliest written account of the golden section occurs in Euclid's derivation in *The Elements*; however, pre-Greek knowledge of ϕ is implicit by its incorporation into the proportions of the Great Pyramid of Egypt, and thus use of the golden section reaches at least as far back into antiquity as the pyramid builders (Livio, 2000; Kappraff, 1991; Ghyka, 1946).

4 Besides this and other geometric properties, ϕ□has a number of other important mathematical properties. It is the positive solution [(1+ 5)/2 = 1.618] to the equation:

(2) $\mathbf{x^2 = x + 1}$

Consequently, the golden section [(-1+ |5)/2 = 0.618≡] is equal to 1 less than the positive solution and is also the negative of the negative solution [(1- ! |5)/2! = !-0.618≡] of this equation.

Substituting from equation (2), ϕ = |(1+ϕ), which can be written as the nested radical:

(3) $$f = 1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1\ldots}}}}$$

and $\phi = 1 + 1/\phi$, which can be written as the continued fraction:

(4) $$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ldots}}}}$$

In addition, because ϕ satisfies the recurrence relation:

(5) $\phi^n = \phi^{n-1} + \phi^{n-2}$

the ratio of successive terms in recurrence sequences (e.g., the Fibonacci series) converges to ϕ as the number of terms approaches infinity. In fact, this very property of the Fibonacci series can be used to approximate ϕ.

defined by the ratio of successive terms in the Fibonacci and other recurrence series as the number of terms approaches infinity.

The successive self-divisibility of ϕ may explain why it appears throughout nature in diverse forms such as the structure of DNA, stages of population growth in rabbits, and plant tissue growth. Regarding the latter, the intersecting phyllotaxis spirals that arrange the florets of pinecones, pineapples, and sunflowers are frequently successive numbers from the Fibonacci series (Kappraff, 1991; Huntley, 1970).[5] Such three-dimensional golden spirals allow efficient packing in growing seed tissue, which maximizes progeny: The corresponding orthogonalization of the leaf divergence angle in growing vegetation maximizes photosynthetic exposure to sunlight and air (Wilson, 1995). Similar economic considerations are important when a three-dimensional form must be preserved during growth, which explains the occurrence of this proportion in the growth of metal crystals (Stevens & Goldman, 1991), the spirals of nautilus shells and ram horns (Ghyka, 1946), and human anatomy (Davis & Altevogt, 1979).

Mathematical elegance, developmental efficiency, and evolutionary pressure are not direct explanations of the psychological preference for ratios in visual objects. In this report, we test and confirm a prediction based on traditional theories of aesthetics in combination with modern form-decomposition theories of object perception: The prediction is that golden rectangles elicit greater perception of depth than other rectangle shapes (Bever, 1987). Our demonstration of this prediction simultaneously confirms the application of classical aesthetic theories to explain the preference for the golden section, and current theories of vision. It is also the first demonstration of the depth-enhancing property of the preferred rectangle; this in turn explains the use of this proportion by nineteenth century American artists to depict realistic landscapes.

Aristotle is credited with two dominant theories of aesthetic satisfaction: *optimal complexity* and *catharsis*. It is significant that *both* aesthetic principles apply to golden rectangles – and both kinds of explanations of the golden rectangle involve analysis of a third dimension and, consequently, depth.

5 Adjacent nucleotide bases are related by a rotation of 36°, an angle of special importance in pentagonal – i.e., golden section – symmetry. The DNA helix is 20 Å in diameter and repeats every 10 residues or every 34 Å. Note the similarity of these two numbers to Fibonacci numbers ...13, 21, 34....

The notion that aesthetic pleasure results from optimal complexity has a history that is both old (Aristotle) and recent (Berlyne) (Berlyne, 1971). The basic premise is that starting in infancy, humans seek an optimal level of complexity – enough to stimulate mental activity, but not so much that it becomes aversive. In the fine arts, this has often been interpreted as optimal novelty, e.g., in our perception of art (Gombrich, 1977) and music (Meyer, 1967). [6] While such investigations are speculative, Aristotle's fundamental insight remains: There is an optimal level of mental complexity associated with aesthetic enjoyment. In the case of golden rectangles, this analysis depends on the idea that visual perception occurs in two stages: an initial stage involves decomposition of a two-dimensional object into canonical geometric forms (Biederman, 1987; Tanaka, 1993), or a preliminary "2-1/2-dimensional" analysis that represents the immediately salient features (Marr, 1982). These are then integrated with object knowledge to access a full representation of an object or scene.

On this view, when humans perceive rectangles, they decompose them into visual primitives, for example, the square, which is the simplest form that underlies rectangles. Thus, for rectangles with ratios of 1:1, 3:2, or 2:1, this perceptual process is either trivial, because it involves only a template match (for a 1:1 rectangle), or relatively simple and finite (for 3:2 or 2:1 rectangles; Figure 1C and 1D in the Appendix). Perceptual processing of a golden rectangle via decomposition into squares results in a square and another golden rectangle 1/ϕ□□the size of the original and rotated 90° (Figure 1A); unlike the decomposition of simpler ratios, this process is indefinite because a golden rectangle always remains after each iteration (1E illustrates this to five iterations). It is the only ratio that replicates itself each time a square is subtracted and is unique in having a recursive analysis in one step each time (Weisstein 2003a, 2003b).[7] The salience of

6 Recently, there have been new attempts to relate this notion of optimal stimulation to optimal levels of endorphin-releasing activity (Vessel & Biederman, 2000).

7 Constructible numbers can be represented by a finite number of arithmetic operations using integers and thus can be constructed with line segments using a straightedge and compass. Thus, a rectangle of 4:3 proportions common to televisions and computer monitors could be constructed by adding four 1:3 rectangles. Transcendental numbers – such as ϕ□– are not constructible (Weisstein, 2003a, 2003b). Note that there is an infinite number of irrational rectangle side ratios that are not easily resolved into constituent squares. However, most such ratios are near one of the easily resolved ratios, e.g., 7:4, 5:4, 4:3, 3:2. On our view, irrational ratios near these ratios are at least temporarily

repeating patterns at the same and different levels of representation has been noted as contributing to aesthetic interest in such fine-art domains as music (Meyer, 1967) and modern mathematical domains as fractals (Livio, 2002). The simultaneous simplicity and completeness of the analyses of the golden rectangle may afford an optimal level of complexity because the same function is used iteratively. Simpler shapes are "uninteresting" because they are completely resolved representationally in a few steps (cf. n.7).

This analysis of the visual processing basis of the preference for the golden rectangle makes an interesting psychophysical prediction: Such rectangles will enhance depth perception. There is a set of four iterative reductions with vanishing points surrounding the center of the figure (Figure 1E). A striking feature of these preliminary visual computational analyses is that, intuitively, *they inculcate the impression of depth*. This is in part based on classic size constancy – a general tendency to represent each square as the same size, which leads to an interpretation of increasing depth near the center. Thus, the optimal complexity analysis of the basis for the preference for golden rectangles predicts that they increase sensitivity to depth perception.[8]

A similar prediction follows from the conflict resolution analysis of the basis for preferring golden rectangles. The roots of conflict-resolution-based aesthetic theories can be traced to Aristotle's account of *catharsis*, in which the aesthetic quality arises from an initial diversity and conflict that is subsequently resolved and unified. Various formulations of this have been proposed, but the general form of these theories is that aesthetic pleasure results from the successful resolution of conflict or uncertainty (Berlyne, 1971).

Visually parsing the golden rectangle with the initially subtracted square on either the left or right side of the rectangle creates an initial perceptual conflict (Figure 1A). This uncertainty occurs because two-dimensional decomposition of the original two- dimensional shape creates overlapping

misperceived as at the nearest available simple ratio. ϕ, however is maximally far from any of the natural ratios, roughly twice as far as any other irrational value (roughly midway between 3:2 and 7:4).

8 Note that the spirals depend on derivation of the reduction via the same rotation. Other reduction schemes exist, such as randomly varying or alternating the rotation direction, which tend to locate the "vanishing" point at the periphery. These patterns are not relevant to our discussion because they are not simple iterations.

squares: The simultaneous left and right parsing of a golden rectangle into squares necessarily results in spatial overlap between the squares (Figure 1F). This initial perceptual conflict in the two-dimensional representation is resolved in a separate three-dimensional representation by perceptual analysis of the two squares as advancing and receding faces of a golden cuboid. Cognitive processing in three-dimensions can result only from the conflict present in the two-dimensional processing; there are no cues of depth present in a golden rectangle itself.[9] Thus, the conflict resolution account of aesthetics combined with the decomposition theory of form-perception can explain how perceptual processing of the golden rectangle differs from perceptual processing of less-preferred forms (e.g., 1:1 and 2:1 rectangles): Form decomposition of a golden rectangle stimulates a representational conflict in two dimensions that can be resolved by accessing a unifying three-dimensional representation.

In sum, the golden rectangle satisfies both classical theories of aesthetics, in combination with a theory of early visual decomposition into canonical shapes (Biederman, 1987; Tanaka, 1993). This confluence of both classic aesthetic principles accounts for the consistent preference of the golden rectangle. In addition, each of the decomposition analyses involves enhancing the salience of a third dimension within the rectangle frame, which makes the prediction that objects and scenes within golden rectangles will elicit enhanced sensitivity to depth.

Literary and artistic considerations of the use of the golden section proportion are consistent with the idea that it enhances depth. According to Benjafield, the golden section in diverse cognitive (and even literary) operations allows people to maximize the distinction between a conceptually *figurative* foreground and background.[10] In the visual domain, the figure-ground relation characteristically involves differences in depth.

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9 In general, it can occur only with rectangles of non-constructible number proportions such as ϕ:1, because geometric primitive parsing of rectangles with constructible number proportions (e.g., 3:2, 2:1) will not result in conflicting representations (Figure 1C and 1D).

10 Humans prefer to divide space or lines into segments with the ϕ□proportion (Rosenthal et al., 2000) and copy bisected lines into a ϕ with greater accuracy than other proportions. ϕ□is the common proportion of positive-negative interpersonal judgments (Watson & Crick, 1965; Vessel & Biederman, 2000), similarity to self-judgments (Svensson, 1977), and positive vs. negative character attributes in Grimm's fairy tales (Benjafield et al., 1980; Benjafield & Green, 1978).

Leonardo da Vinci and his contemporary Renaissance painters made frequent use of the golden section for this purpose (Livio, 2002). We have noted that nineteenth century American artists characteristically used the golden section ratio for framing landscapes. Schools of landscape painters centuries earlier may also have been aware of the depth enhancing property of the golden rectangle. For example, in a famous Flemish painting of a seventeenth century exhibition of 39 pictures (Plate 1), the mean long- to short-side ratio of the landscapes is 1.6:1, unlike the ratios in other kinds of scenes, which averaged 1.4:1.[11]

It is important to complement such anecdotal aesthetic and artistic evidence with experimental studies of the perception of depth in golden section frames. If depth cues are already present in a scene, then framing it in a golden rectangle should enhance these cues. This hypothesis is also falsifiable: Both variants of the aesthetic analysis predict that depth will be enhanced in the center of the golden section frame, while depth is homogenous in other frame shapes. We tested these predictions experimentally with two perceptual judgment tasks that required participants to judge if stimuli of varying sizes appeared to be at the same depth or at different depths.

We used two illusions in separate experiments. In a *chromostereopsis* illusion (Figure 2A), red and blue stimuli are perceived as advancing and receding in depth, respectively, when identical in size and on a black background. In the *Helmholtz* illusion (Figure 2B), gray stimuli of identical size appear to advance and recede, when viewed on a black and white background, respectively.

11 We excluded six of the paintings (two landscapes and four interiors), which were visually angled 45° from a head-on view, because of the difficulty in calculating the exact frame ratio. It may be significant that this epoch of Dutch and Flemish Renaissance painting involved the then novel goal of painting landscapes as objects in their own right. The novelty of patron-supported landscapes and portraits may have left it up to the artist to choose the frame, while such choices became more constrained in later centuries as this kind of fine-art became more conventional. Similarly, nineteenth century American artists were the first in modern times to paint nature directly, breaking conventional studio methods, with greater freedom to choose frame proportions.

Plate 1: Cognoscenti in a Room hung with Pictures
Flemish, c. 1620, oil on oak, 95.9 x 123.5 cm

In our studies, we varied the physical sizes of the stimuli within a pair as well as the correspondence between the depth cue of physical size and the illusory depth cue.[12] When stimuli physically differ in size, the larger is usually seen as closer. In half of our trials, the illusory cues corresponded to and thus *exaggerated* the perception of depth, and in the other half of the trials, the illusory cues conflicted with and thus *attenuated* the perception of depth. This design property of the experimental paradigm allowed us to measure if participants did indeed perceive illusory depth and, crucially, if perceived illusory depth varied as a function of the frame shape through which they viewed the stimuli (Figure 3). Stimuli were presented in frames of equal area but with three different rectangular shapes: long to short side ratios of 1:1, ϕ:1, and 2:1.[13] Participants' depth

12 Size varied from 0–20% in 5% increments.

13 All frames were presented centered on the computer screen. Each of the two stimuli was presented centered in the hypothetical square made by visually parsing a golden rectangle into a square on the left or right side, regardless of the actual frame shape that surrounded the stimuli. This procedure ensured that stimuli were presented in identical positions for each of the three frames. Stimuli and frames were presented on a 14-in. (35.6-cm)

judgments were recorded for each of these frames to measure their perception of illusory depth.[14, 15]

Overall, illusory depth perception was about 16% larger for stimuli presented in golden rectangles than in non-golden rectangles. Assessing the reliability of this effect involved two measures. First, we computed the difference in depth judgments between exaggerating illusory trials (e.g., the larger of the two circles was red) and attenuating illusory trials (e.g., the larger of the two circles was blue) (Figure 3A). Second, this contrast score

display at 640 ∞ 480 (69 dpi) resolution. Frames varied in shape (283 ∞ 283; 361 ∞ 223; and 401 ∞ 200 pixels), but were equal (100 cm^2) in area. Stimuli varied, but averaged two cm^2 in area. Note that the physical frame of the computer display was 4:3, a factor that on our theory might work against any overall perception of depth within the display.

14 In both experiments, for each participant, we examined trials in which the stimuli within a pair physically differed (80% of the trials) and measured the proportion of different depth judgments for each of four conditions: trial type (illusory vs. control trials) crossed with depth cue correspondence (attenuating vs. exaggerating). This measure reflects the accuracy of participants' perceptions (i.e., how often they perceived differently sized stimuli as different in depth) and allowed us to establish that illusory depth altered (enhanced) participants' perceptions regardless of frame shape. For each these four conditions in both experiments, we computed 95% confidence intervals corrected for continuity on the mean proportion of different depth judgments ($n = 24$).

15 For both illusions, on control trials (e.g., when stimuli were both red or both blue), and on attenuating illusory trials, participants judged differently sized stimuli to be of different depths at chance (50%; all ps > .05). However, for both the chromostereopsis and Helmholtz illusions, on illusory trials that exaggerated size differences, participants judged physical differences as different depth differences, 73.1% and 84.2% of the time, respectively. For both illusions, the number of different judgments for the exaggerating illusory trials was both greater than chance (all ps < .05) and greater than either the attenuating illusory (all ps < .05) or control conditions (all ps < .05). Thus, in both illusions, illusory cues that exaggerated physical (size) cues enhanced depth perception by improving participant's depth judgments from chance (i.e., guessing) performance. These results may be interesting in their own right: They suggest that it is possible to enhance depth by illusions that move out from the scene towards the viewer, but not possible to enhance depth via illusions that would depend on moving back away from the viewer. This may reflect an important property of viewing two-dimensional images, but is orthogonal to the main concern of this paper.

We present the results of the non-parametric Wilcoxon signed rank test for paired observations on these contrast scores in the main text, because the contrast scores were not normally distributed for the non-golden rectangle conditions. We also performed an analysis of variance on these contrast scores, which we present here for completeness: This analysis, which assumes data are normally distributed, revealed a significant main effect of frame shape across both illusions, $F(1,46)=5.82$, $MS_e=133.96$, $p=.02$, $r=.34$.

(Rosenthal et al., 2000) was computed for each participant for both ϕ:1 and non-ϕ:1 frame shapes (Figure 3B). Analysis of these scores revealed a consistent enhancement of the depth illusion in the golden section (ϕ:1) shaped frame compared with the other two frame shapes, across both illusions, Z=-2.26, p=.012 and individually for each illusion (chromostereopsis, Z=-1.66, p = .049; Helmholtz, Z=-1.66, p = .049; Wilcoxon signed rank test) (see n. 15).

Both aesthetic explanations of the golden rectangle predict that the depth- enhancement due to ϕ will be largest in the center of the rectangle: The optimal complexity explanation sets up four spirals whose vanishing points cluster around the center and the conflict resolution explanation depends on a two-dimensional overlap occurring in the center. To test this, we used the color illusion, placing the stimuli on the left, right or center of the ϕ:1 and the 2:1 rectangles. We analyzed it in the same way as the other studies, contrasting the depth enhancement against the depth inhibition (Table 2).[16] For the golden rectangle, the illusory depth perception contrast score was 48% more for stimuli presented in the central position than outside the central position, $F(1, 46)$=10.56, MS_e=.009, p=.0018, r=56. In contrast, for the 2:1 rectangle, there was only a 1% difference due to lateral position (F<1). This difference gives further support not only to the fact that the golden rectangle enhances depth perception, but also supports the aesthetic and visual interpretations of the basis for this enhancement.[17]

16 Using the chromostereopsis illusion, we presented the stimuli in the center of a golden rectangle or outside the central position and inside the hypothetical left or right square that a golden rectangle could be decomposed into. In a separate experiment, stimuli in identical positions were presented inside a 2:1 rectangle. We analyzed the results in the same way as in the other studies, contrasting the depth enhancement against the depth inhibition. Participants viewed stimuli in either a golden rectangle (n = 24) or a 2:1 rectangle (n = 24). 95% confidence intervals for the simple effect of lateral position (central vs. lateral) for both frame shapes = ± 6.3% (i.e., differences within a frame shape > 6.3% are statistically significant, p < .05).

17 In both experiments n = 24. The 1:1 rectangle was not tested, because positioning the stimuli to left and right would have resulted in them appearing outside the rectangle. Note that across the two frame types, the central position did elicit 12% more depth for participants who saw only the golden rectangle than for participants who saw only the 2:1 rectangle. This numerical replication of the fact that the golden rectangle enhances depth did not reach statistical significance because of across- subject variance.

Table 2: Depth illusions are larger for stimuli centered in golden rectangles (but not 2:1 rectangles), a prediction of both the *optimal complexity* and *conflict resolution* aesthetic theories.

	Mean illusory depth perception contrast scores (%)	
	Frame Shape	
Position	ϕ:1	2.1
Central	27.5	24.6
Lateral	18.6	24.3

These novel results confirm the combination of traditional aesthetic theories and modern two-stage visual models to explain the aesthetic preference for golden rectangles. The optimal complexity and perceptual conflict resolution analyses together account for the strong preference for the golden rectangle: Both aesthetic analyses involve accessing a third dimension, which accounts for the enhancement of depth in the golden rectangle frame, especially in its center. Nineteenth century American landscape painters, who were concerned to emphasize broad expanses of American geography, selectively framed their landscapes using golden section proportions. This suggests that they intuitively knew that this choice would increase the perceived depth of their two-dimensional paintings and thereby depict American landscapes in the most impressive way.

3 Conclusion: Universals of the Third Kind in Cognition and Language

In their recent influential paper, Chomsky and colleagues (Hauser, Chomsky, & Fitch, 2002) differentiate at least three major types of linguistic universals, i) Basic principles of Universal Grammar (UG); ii) Universals resulting from language input/output interfaces; iii) Universals that result from the action of mathematical/physical laws on computational systems. An important feature of type (iii) universals is that their existence is not directly coded either as part of UG, nor do they result from interface constraints; rather, they are an automatic result of the existence of the laws, and how those laws naturally constrain the form of languages. The effect of the Fibonacci series discussed in the first section of this paper is an example of how a natural law can result in picking out optimal computational features of languages. The research in this paper has raised the possibility of a similar result in the case of vision, aesthetics, and the perception of depth. We start with three empirical assumptions: i) an early stage of visual analysis is decomposition of simple figures into basic ones; ii) the two principles of aesthetic enjoyment are ideal complexity and implicit problem solving; iii) representations of overlapping figures creates an illusion of depth; iv) identical figures of different sizes create an illusion of depth. Given these empirical assumptions, the Golden Mean rectangle will be preferred by principles (i) and (ii) and will enhance the perception of depth within its space by assumptions (iii) and (iv). This renders the effects of the Golden Mean automatic, and totally transparent. While assumptions (iii) and (iv) are long established, we expect that further research will increase the support for assumptions (i) and (ii), and thereby for our analysis in general.

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Bionotes

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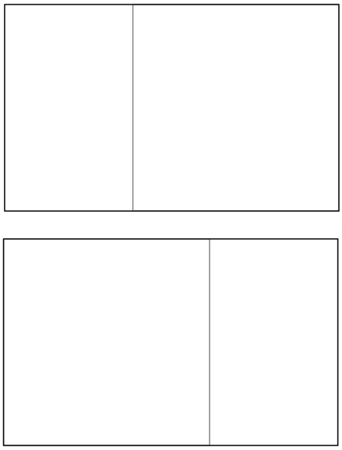
Appendix

A



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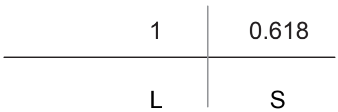

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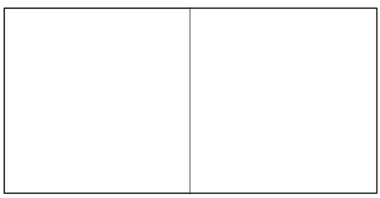

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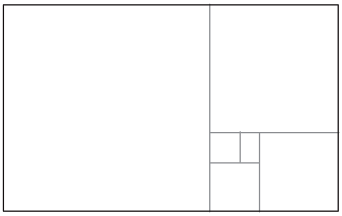

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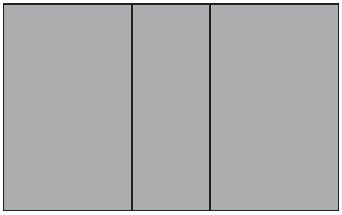

Figure 1

A A *golden rectangle* is an aesthetically preferred visual-form.
B It can be constructed from the *golden section*, defined by division of a line or space into short (*S*) and long (*L*) segments such that the ratio of the shorter segment to the longer segment is the same as the ratio of the longer segment to the sum of both segments. There is a unique point of division for this mathematical problem: If $L = 1$, then S is an irrational number, ϕ, with the value 1.618.... Likewise, if $L+S=1$, the L is the golden section ($\phi-1$), 0.618.... A golden rectangle (A), which has proportions of ϕ:1, is unique among rectangles in that when parsed geometrically into a square of length (*L*), the shape of the area remaining is identical to the original shape (i.e., another golden rectangle of *L* ∞□*S* dimensions and thus 0.618... the size of the original). Theories of vision assume that rectangles (and other shapes) are decomposed into visual primitives such as squares. This process is simple and finite for rectangles of 3/2:1
C or 2:1
D proportions, but is indefinite for golden rectangles
E (shown for five iterations of decomposition into a square). Aristotle's two classical aesthetic theories each offer an explanation of the preference for golden rectangles. On the *optimal complexity* view, the golden section elicits iterative decomposition, re-applying the same analysis to successively smaller golden section shapes (E). Note that the successively smaller squares elicit the perception of depth via size constancy, which

makes the smaller squares seem more distant. On the *conflict resolution* aesthetic theory, perceptual conflict can be created when a golden rectangle is decomposed into the visual primitive of the square by the ambiguity present

F in parsing the square on either the right or the left side (A). This perceptual conflict in two dimensions is resolved by accessing an additional three-dimensional representation. Thus, integrating the form-decomposition theory of vision with the two classical theories of aesthetics can account for *why* humans prefer this particular visual-form. These explanations also predict that golden rectangles (A) will elicit greater perception of depth than non-golden rectangles such as 2:1 rectangles (D) or squares, which do not result in a successive iterations of decomposition and perceptual conflict.

A

B

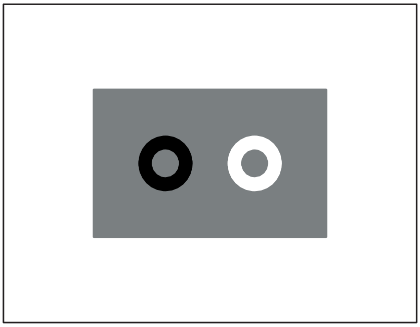

Figure 2

A Visual illusions of depth viewed within golden rectangle frames. In the *chromostereopsis* illusion, a red circle appears closer to the viewer than a blue circle when they are of identical size against a black background.

B In the *Helmholtz* illusion, a gray circle on a black disk appears closer to the viewer than a gray circle on a white disk when the circles are of identical size.

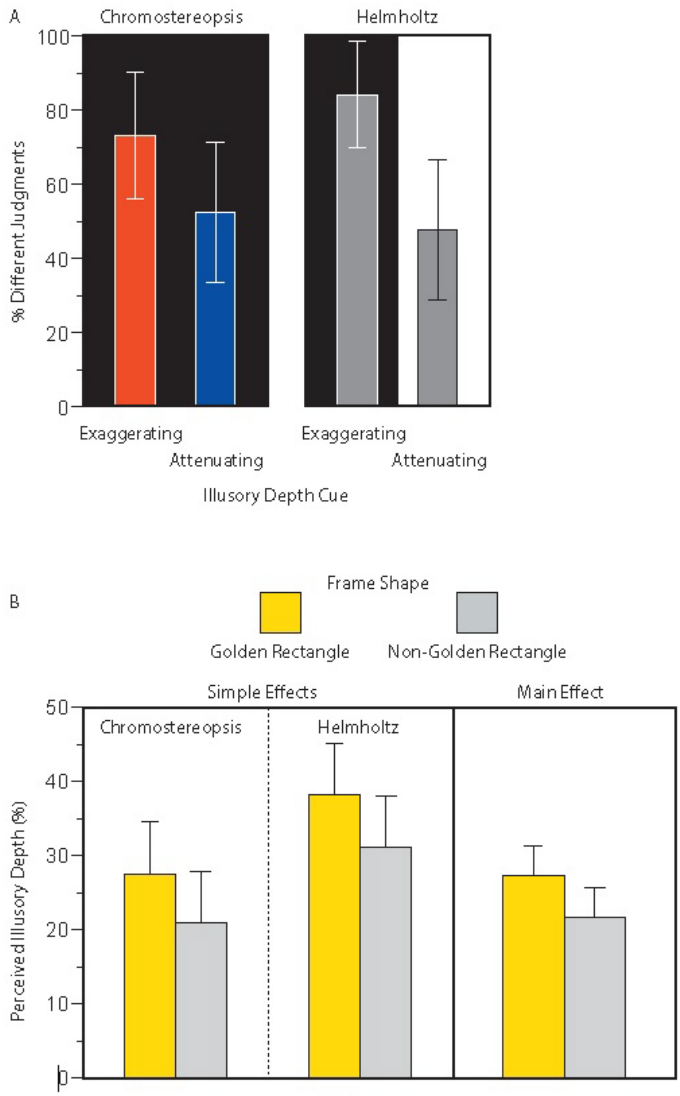

Figure 3

Depth illusions are larger in golden rectangles than in other rectangle shapes. **A** First we established that our participants perceived the depth illusions overall. In separate experiments using the chromostereopsis (Left) and Helmholtz illusions (Right), for each participant, we examined trials in which the stimuli within a pair physically differed (80% of the trials) and measured the proportion of different depth judgments for each of four conditions. On *illusory* trials, stimuli differed in color (e.g., one red and one blue circle), and the visual illusion could either *exaggerate* or *attenuate* physical size cues, depending on the color of the larger of the two circles. On *control* trials, stimuli also differed in size, but were of identical color (e.g., both red or both blue). In addition, in both experiments, the proportion of "different" depth judgments was larger for trials with exaggerating cues than for attenuating cues for illusory trials (both ps<.05), but not control trials (not shown; both ps>.05). These results confirm that our participants perceived illusory depth.

B The depth illusion is larger for golden rectangle frames. We performed a Wilcoxon signed rank test on computed difference scores between the two kinds of illusory trials (exaggerating and attenuating) for each illusion for *golden rectangle* (f:1) and *non-golden rectangle* (2:1 and 1:1) frame shapes. Bars indicate participant means for these scores; error bars indicate 95% confidence intervals for the simple and main effects of frame shape. Critically, there was a significant main effect of frame shape on participants' perception of illusory depth across both experiments (p<.05): Participants' illusory depth perception was 16% larger in golden-rectangle than in non-golden rectangle frame shapes.